



International Journal of Innovative Research in Science Engineering and Technology (IJIRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)



Impact Factor: 8.699

Volume 14, Issue 8, August 2025

Interval-Valued Bipolar Vague Resolvable and Irresolvable Spaces

F. Prishka¹, Dr. L. Mariapresenti²

Research Scholar, Department of Mathematics, Nirmala College for Women, Redfield's, Coimbatore,
Tamil Nadu, India¹

Assistant Professor, Department of Mathematics, Nirmala College for Women, Redfield's, Coimbatore,
Tamil Nadu, India²

ABSTRACT: An innovative idea of an Interval-Valued bipolar vague resolvable spaces, Interval-Valued bipolar vague irresolvable spaces, Interval-Valued bipolar vague almost resolvable spaces and an Interval-Valued almost irresolvable spaces are presented and their belongings are carefully well-thought-out and inspected. Some of the thought-provoking propositions based on the newly introduced set are attainable.

KEYWORDS: Interval-Valued bipolar vague sets, Interval-Valued bipolar vague topology, Interval-Valued- bipolar vague resolvable spaces, Interval-Valued bipolar vague irresolvable spaces, Interval-Valued almost resolvable spaces and an Interval-Valued almost irresolvable spaces.

I. INTRODUCTION

Ambiguous set was Obtainable by L.A.Zadeh [11] in 1965. The knowledge of fuzzy topology was proclaimed by C.L.Chang [3] in 1968. The generalized closed sets in general topology persisted primarily by N.Levine [9] in 1970. K.Atanassov [2] in 1986 existing the insight of intuitionistic fuzzy sets. The belief of vague set theory was publicized by W.L.Gau and D.J.Buehrer [7] in 1993. D.Coker [6] in 1997 proclaimed intuitionistic fuzzy topological spaces. Bipolar- valued fuzzy sets, which was existing by K.M.Lee [8] in 2000 is a scope of fuzzy sets whose membership degree range is enlarged from the Interval-Valued $[0, 1]$ to $[-1, 1]$. An innovative class of generalized bipolar vague sets was declared by S.Cicily Flora and I.Arockiarani [4] in 2016. F.Prishka and L.Mariapresenti [10] introduced the concepts of Interval-Valued bipolar vague Volterra spaces in 2025. The perseverance of this paper is too present-day and learning the thought of Interval-Valued bipolar vague resolvable spaces, Interval-Valued bipolar vague irresolvable spaces, Interval-Valued almost resolvable spaces and an Interval-Valued almost irresolvable spaces.

II. PRELIMINARIES

Here in this paper the bipolar vague topological spaces are denoted by (S, BV_{τ}) . Also, the bipolar vague interior, bipolar vague closure of a bipolar vague set N are denoted by $BVInt(N)$ and $BVCl(N)$. The complement of a bipolar vague set N is denoted by N^c and the empty set and whole sets are denoted by 0_{\sim} and 1_{\sim} respectively.

Definition 2.1: [8] Let S be the universe. Then a bipolar valued fuzzy sets, N on S is defined by positive membership function μ_N^+ , that is $\mu_N^+: S \rightarrow [0, 1]$, and a negative membership function μ_N^- , that is $\mu_N^-: S \rightarrow [-1, 0]$. For the sake of simplicity, we shall use the symbol

$$N = \{ \langle s, \mu_N^+(s), \mu_N^-(s) \rangle : s \in S \}.$$

Definition 2.2: [8] Let N and M be two bipolar valued fuzzy sets then their union, intersection and complement are defined as follows:

- (i) $\mu_{N \cup M}^+ = \max \{ \mu_N^+(s), \mu_M^+(s) \}$
- (ii) $\mu_{N \cup M}^- = \min \{ \mu_N^-(s), \mu_M^-(s) \}$
- (iii) $\mu_{N \cap M}^+ = \min \{ \mu_N^+(s), \mu_M^+(s) \}$
- (iv) $\mu_{N \cap M}^- = \max \{ \mu_N^-(s), \mu_M^-(s) \}$

(v) $\mu_{N^c}^+(s) = 1 - \mu_N^+(s)$ and $\mu_{N^c}^-(s) = -1 - \mu_N^-(s)$ for all $s \in S$.

Definition 2.3: [7] A vague set N in the universe of discourse T is a pair of (t_N, f_N) where

$t_N: U \rightarrow [0, 1]$, $f_N: U \rightarrow [0, 1]$ are the mapping such that $t_N + f_N \leq 1$ for all $t \in T$. The function t_N and f_N are called true membership function and false membership function respectively. The Interval-Valued $[t_N, 1 - f_N]$ is called the vague value of t in N , and denoted by $v_N(t)$, that is

$$v_N(t) = [t_N(t), 1 - f(t)].$$

Definition 2.4: [7] Let A be a non-empty set and the vague set N and M in the form

$N = \{ \langle s, t_N(s), 1 - f_N(s) \rangle : s \in S \}$, $B = \{ \langle s, t_M(s), 1 - f_M(s) \rangle : s \in S \}$. Then

- (i) $N \subseteq M$ if and only if $t_N(s) \leq t_M(s)$ and $1 - f_N(s) \leq 1 - f_M(s)$
- (ii) $N \cup M = \{ \langle \max(t_N(s), t_M(s)), \max(1 - f_N(s), 1 - f_M(s)) \rangle / s \in S \}$.
- (iii) $N \cap M = \{ \langle \min(t_N(s), t_M(s)), \min(1 - f_N(s), 1 - f_M(s)) \rangle / s \in S \}$.
- (iv) $N^c = \{ \langle s, f_N(s), 1 - t_N(s) \rangle : s \in S \}$.

Definition 2.5: [1] Let S be the universe of discourse. A bipolar-valued vague set N in S is an object having the form $N = \{ \langle s, [t_N^+(s), 1 - f_N^+(s)], [-1 - f_N^-(s), t_N^-(s)] \rangle : s \in S \}$ where $[t_N^+, 1 - f_N^+] : S \rightarrow [0, 1]$ and $[-1 - f_N^-, t_N^-] : S \rightarrow [-1, 0]$ are the mapping such that $t_N^+(s) + f_N^-(s) \leq 1$ and $-1 \leq t_N^- + f_N^+$. The positive membership degree $[t_N^+(s), 1 - f_N^+(s)]$ denotes the satisfaction region of an element s to the property corresponding to a bipolar-valued set N and the negative membership degree $[-1 - f_N^-(s), t_N^-(s)]$ denotes the satisfaction region of s to some implicit counter property of N . For a sake of simplicity, we shall use the notion of bipolar vague set $v_N^+ = [t_N^+, 1 - f_N^+]$ and $v_N^- = [-1 - f_N^-, t_N^-]$.

Definition 2.6: [5] A bipolar vague set $N = [v_N^+, v_N^-]$ of a set T with $v_N^+ = 0$ implies that $t_N^+ = 0$, $1 - f_N^+ = 0$ and $v_N^- = 0$ implies that $t_N^- = 0$, $-1 - f_N^- = 0$ for all $s \in T$ is called zero bipolar vague set and it is denoted by 0.

Definition 2.7: [5] A bipolar vague set $N = [v_N^+, v_N^-]$ of a set T with $v_N^+ = 1$ implies that $t_N^+ = 1$, $1 - f_N^+ = 1$ and $v_N^- = -1$ implies that $t_N^- = -1$, $-1 - f_N^- = -1$ for all $s \in T$ is called unit bipolar vague set and it is denoted by 1.

Definition 2.8: [4] Let $N = \langle s, [t_N^+, 1 - f_N^+], [-1 - f_N^-, t_N^-] \rangle$ and $M = \langle s, [t_M^+, 1 - f_M^+], [-1 - f_M^-, t_M^-] \rangle$ be two bipolar vague sets then their union, intersection and complement are defined as follows:

- (i) $N \cup M = \{ \langle s, [t_{N \cup M}^+(s), 1 - f_{N \cup M}^+(s)], [-1 - f_{N \cup M}^-(s), t_{N \cup M}^-(s)] \rangle / s \in S \}$ where
 $t_{N \cup M}^+(s) = \max \{ t_N^+(s), t_M^+(s) \}$, $t_{N \cup M}^-(s) = \min \{ t_N^-(s), t_M^-(s) \}$ and
 $1 - f_{N \cup M}^+(s) = \max \{ 1 - f_N^+(s), 1 - f_M^+(s) \}$,
 $-1 - f_{N \cup M}^-(s) = \min \{ -1 - f_N^-(s), -1 - f_M^-(s) \}$.
- (ii) $N \cap M = \{ \langle s, [t_{N \cap M}^+(s), 1 - f_{N \cap M}^+(s)], [-1 - f_{N \cap M}^-(s), t_{N \cap M}^-(s)] \rangle / s \in S \}$ where
 $t_{N \cap M}^+(s) = \min \{ t_N^+(s), t_M^+(s) \}$, $t_{N \cap M}^-(s) = \max \{ t_N^-(s), t_M^-(s) \}$ and
 $1 - f_{N \cap M}^+(s) = \min \{ 1 - f_N^+(s), 1 - f_M^+(s) \}$,
 $-1 - f_{N \cap M}^-(s) = \max \{ -1 - f_N^-(s), -1 - f_M^-(s) \}$.
- (iii) $N^c = \{ \langle s, [f_N^+(s), 1 - t_N^+(s)], [-1 - t_N^-(s), f_N^-(s)] \rangle / s \in S \}$.

Definition 2.9: [4] Let N and M be two bipolar vague sets defined over a universe of discourse S . We say that $N \subseteq M$ if and only if $t_N^+(s) \leq t_M^+(s)$, $1 - f_N^+(s) \leq 1 - f_M^+(s)$ and $t_N^-(s) \geq t_M^-(s)$, $-1 - f_N^-(s) \geq -1 - f_M^-(s)$ for all $s \in S$.

Definition 2.10: [4] A bipolar vague topology (BVT) on a non-empty set S is a family BV_τ of bipolar vague set in S satisfying the following axioms:

- (i) $0, 1 \in BV_\tau$
- (ii) $G_1 \cap G_2 \in BV_\tau$, for any $G_1, G_2 \in BV_\tau$
- (iii) $\cup G_i \in BV_\tau$, for any arbitrary family $\{G_i : G_i \in BV_\tau, i \in I\}$.

In this case the pair (S, BV_τ) is called a bipolar vague topological space and any bipolar vague set (BVS) in BV_τ is known as bipolar vague open set in S . The complement N^c of a bipolar vague open set (BVOS) A in a bipolar vague topological space (S, BV_τ) is called a bipolar vague closed set (BVCS) in N .

Definition 2.11: [4] Let (S, BV_τ) be a bipolar vague topological space

$N = \langle s, [t_N^+, 1 - f_N^+], [-1 - f_N^-, t_N^-] \rangle$ be a bipolar vague set in N . Then the bipolar vague interior and bipolar vague closure of N are defined by,

$$BVInt(N) = \cup \{R: R \text{ is a bipolar vague open set in } N \text{ and } R \subseteq N\},$$

$$BVCl(N) = \cap \{W: W \text{ is a bipolar vague closed set in } S \text{ and } N \subseteq W\}.$$

Note that $BVCl(N)$ is a bipolar vague closed set and $BVInt(N)$ is a bipolar vague open set in S . Further,

- (i) N is a bipolar vague closed set in S if and only if $BVCl(N) = N$,
- (ii) N is a bipolar vague open set in S if and only if $BVInt(N) = N$.

Definition 2.12: [4] Let (S, BV_τ) be a bipolar vague topological space. A bipolar vague set N in (S, BV_τ) is said to be a generalized bipolar vague closed set if $BVCl(N) \subseteq R$ whenever $N \subseteq R$ and R is bipolar vague open. The complement of a generalized bipolar vague closed set is generalized bipolar vague open set.

Definition 2.13: [4] Let (S, BV_τ) be a bipolar vague topological space and N be a bipolar vague set in N . Then the generalized bipolar vague closure and generalized bipolar vague interior of N are defined by,

$$GBVCl(N) = \cap \{R: R \text{ is a generalized bipolar vague closed set in } N \text{ and } N \subseteq R\},$$

$$GBVInt(N) = \cup \{R: R \text{ is a generalized bipolar vague open set in } N \text{ and } N \supseteq R\}.$$

Definition 2.14: [10] An Interval-Valued bipolar vague sets \widetilde{B}^{BV} over a universe of discourse H is defined as an object of the form $\widetilde{B}^{BV} = \{ \langle h_i, [[t_{\widetilde{B}^{BV}}^+(h_i), 1 - f_{\widetilde{B}^{BV}}^+(h_i)], [-1 - f_{\widetilde{B}^{BV}}^-(h_i), t_{\widetilde{B}^{BV}}^-(h_i)] \rangle : h_i \in H \}$ where $[t_{\widetilde{B}^{BV}}^+, 1 - f_{\widetilde{B}^{BV}}^+]: H \rightarrow D[0,1]$ and $[-1 - f_{\widetilde{B}^{BV}}^-, t_{\widetilde{B}^{BV}}^-]: H \rightarrow D[-1,0]$ are the mapping such that $t_{\widetilde{B}^{BV}}^+ + f_{\widetilde{B}^{BV}}^+ \leq 1$ and $-1 \leq t_{\widetilde{B}^{BV}}^- + f_{\widetilde{B}^{BV}}^-$. The positive membership degree $[t_{\widetilde{B}^{BV}}^+(h_i), 1 - f_{\widetilde{B}^{BV}}^+(h_i)]$ denotes the satisfaction region of an element h_i to the property corresponding to an Interval-Valued bipolar-valued set \widetilde{B}^{BV} and the negative membership degree $[-1 - f_{\widetilde{B}^{BV}}^-(h_i), t_{\widetilde{B}^{BV}}^-(h_i)]$ denotes the satisfaction region of h_i to some implicit counter property of \widetilde{B}^{BV} . For a sake of simplicity, we shall use the notion of Interval-Valued bipolar vague set $v_{\widetilde{B}^{BV}}^+ = [t_{\widetilde{B}^{BV}}^+, 1 - f_{\widetilde{B}^{BV}}^+]$ and $v_{\widetilde{B}^{BV}}^- = [-1 - f_{\widetilde{B}^{BV}}^-, t_{\widetilde{B}^{BV}}^-]$.

Definition 2.15: [10] An Interval-Valued bipolar vague topology (IBVT) on H is a family IBV_τ of Interval-Valued bipolar vague sets (IBVS) in H satisfying the following axioms:

- (i) $0_\sim, 1_\sim \in IBV_\tau$
- (ii) $T_1 \cap T_2$ for any $T_1, T_2 \in IBV_\tau$
- (iii) $\cup T_i \in IBV_\tau$, for any family $\{T_i: i \in J\} \subseteq IBV_\tau$

In this case the pair (H, IBV_τ) is called an Interval-Valued bipolar vague topological space (IBVTS) and any Interval-Valued bipolar vague set (IBVS) in IBV_τ is known as an Interval-Valued bipolar vague open set (IBVOS) in H . The complement of an Interval-Valued bipolar vague open set N^c in an Interval-Valued bipolar vague topological space (H, IBV_τ) is called an Interval-Valued bipolar vague closed set (IBVCS) in H .

Definition 2.16: [10] Let $H = \langle h, [[t_M^{+L}(h), t_M^{+U}(h)][1 - f_M^{+L}(h), 1 - f_M^{+U}(h)], [-1 - f_M^{-L}(h), -1 - f_M^{-U}(h)][t_M^{-L}(h), t_M^{-U}(h)]] \rangle$ and $M = \langle h, [[t_C^{+L}(h), t_C^{+U}(h)][1 - f_C^{+L}(h), 1 - f_C^{+U}(h)], [-1 - f_C^{-L}(h), -1 - f_C^{-U}(h)][t_C^{-L}(h), t_C^{-U}(h)]] \rangle$ be two Interval-Valued bipolar vague sets then their union, intersection and complement are defined as follows:

- (i) $M \cup O = \{ \langle h, [[t_{M \cup O}^{+L}(h), t_{M \cup O}^{+U}(h)][1 - f_{M \cup O}^{+L}(h), 1 - f_{M \cup O}^{+U}(h)], [-1 - f_{M \cup O}^{-L}(h), -1 - f_{M \cup O}^{-U}(h)][t_{M \cup O}^{-L}(h), t_{M \cup O}^{-U}(h)]] \rangle : h \in H \}$ where

$$t_{M \cup O}^{+L}(h) = \max \{t_M^{+L}(h), t_C^{+L}(h)\},$$

$$t_{M \cup O}^{+U}(h) = \max \{t_M^{+U}(h), t_C^{+U}(h)\},$$

$$t_{M \cup O}^{-L}(h) = \min \{t_M^{-L}(h), t_C^{-L}(h)\} \text{ and}$$

$$t_{M \cup O}^{-U}(h) = \min \{t_M^{-U}(h), t_C^{-U}(h)\}$$

$$1 - f_{M \cup O}^{+L}(h) = \max \{1 - f_M^{+L}(h), 1 - f_C^{+L}(h)\},$$

$$1 - f_{M \cup O}^{+U}(h) = \max \{1 - f_M^{+U}(h), 1 - f_C^{+U}(h)\},$$

$$-1 - f_{M \cup O}^{-L}(h) = \min \{-1 - f_M^{-L}(h), -1 - f_C^{-L}(h)\} \text{ and}$$

$$\begin{aligned}
 & -1 - f_{M \cup O}^{-U}(h) = \min \{-1 - f_M^{-U}(h), -1 - f_O^{-U}(h)\}. \\
 \text{(ii)} \quad & M \cap O = \{h, [t_{M \cap O}^{+L}(h), t_{M \cap O}^{+U}(h)][1 - f_{M \cap O}^{+L}(h), 1 - f_{M \cap O}^{+U}(h)], [-1 - f_{M \cap O}^{-L}(h), -1 - f_{M \cap O}^{-U}(h)][t_{M \cap O}^{-L}(h), t_{M \cap O}^{-U}(h)]\} / h \in H \text{ where} \\
 & \quad t_{M \cap O}^{+L}(h) = \min \{t_M^{+L}(h), t_O^{+L}(h)\}, \\
 & \quad t_{M \cap O}^{+U}(h) = \min \{t_M^{+U}(h), t_O^{+U}(h)\}, \\
 & \quad t_{M \cap O}^{-L}(h) = \max \{t_M^{-L}(h), t_O^{-L}(h)\} \text{ and} \\
 & \quad t_{M \cap O}^{-U}(h) = \max \{t_M^{-U}(h), t_O^{-U}(h)\} \\
 & \quad 1 - f_{M \cap O}^{+L}(h) = \min \{1 - f_M^{+L}(h), 1 - f_O^{+L}(h)\}, \\
 & \quad 1 - f_{M \cap O}^{+U}(h) = \min \{1 - f_M^{+U}(h), 1 - f_O^{+U}(h)\}, \\
 & \quad -1 - f_{M \cup O}^{-L}(h) = \max \{-1 - f_M^{-L}(h), -1 - f_O^{-L}(h)\} \text{ and} \\
 & \quad -1 - f_{M \cup O}^{-U}(h) = \max \{-1 - f_M^{-U}(h), -1 - f_O^{-U}(h)\}. \\
 \text{(iii)} \quad & M^c = \{[h, [f_M^{+L}(h), f_M^{+U}(h)][1 - t_M^{+L}(h), 1 - t_M^{+U}(h)][-1 - t_M^{-L}(h), -1 - t_M^{-U}(h)][f_M^{-L}(h), f_M^{-U}(h)]]\} / h \in H\}.
 \end{aligned}$$

Definition 2.17: [10] Let (H, IBV_τ) be an Interval-Valued bipolar vague topological space and $B = \{h, [t_M^{+L}(h), t_M^{+U}(h)][1 - f_M^{+L}(h), 1 - f_M^{+U}(h)], [-1 - f_M^{-L}(h), -1 - f_M^{-U}(h)][t_M^{-L}(h), t_M^{-U}(h)]\}$ be an Interval-Valued bipolar vague set in Y . Then the Interval-Valued bipolar vague interior and an Interval-Valued bipolar vague closure defined by,

- (i) $IBVInt(M) = \cup \{D: D \text{ is an Interval-Valued bipolar vague open set in } H \text{ and } D \subseteq M\},$
(ii) $IBVC(M) = \cap \{P: P \text{ is an Interval-Valued bipolar vague closed set in } H \text{ and } M \subseteq P\}.$

Note that for any Interval-Valued bipolar vague set H in (H, IBV_τ) , we have $IBVC(M^c) = (IBVInt(M))^c$ and $IBVInt(M^c) = (IBVC(M))^c$ and $IBVC(M)$ is an Interval-Valued bipolar vague closed set and $IBVInt(M)$ is an Interval-Valued bipolar vague open set in H . Further we have, if M is an Interval-Valued bipolar vague closed set in H , then $IBVC(M) = M$ and if M is an Interval-Valued bipolar vague open set in H , then $IBVInt(M) = M$.

Definition 2.18: [10] An Interval-Valued bipolar vague set M in an Interval-Valued bipolar vague topological space (H, IBV_τ) is called an Interval-Valued bipolar vague dense if there exists no Interval-Valued bipolar vague closed set O in (H, IBV_τ) such that $M \subset O \subset 1_\sim$.

Definition 2.19: [10] An Interval-Valued bipolar vague set M in an Interval-Valued bipolar vague topological space (H, IBV_τ) is called an Interval-Valued bipolar vague nowhere dense set if there exists no Interval-Valued bipolar vague open set O in (H, IBV_τ) such that $O \subset IBVC(M)$. That is $IBVInt(IBVC(M)) = 0_\sim$.

Definition 2.20: [10] An Interval-Valued bipolar vague topological space (H, IBV_τ) is called an Interval-Valued bipolar vague first category set if $M = \cap_{i=1}^\infty M_i$, where M_i 's are Interval-Valued bipolar vague nowhere dense set in (H, IBV_τ) . Any other Interval-Valued bipolar vague set in (H, IBV_τ) is said to be an Interval-Valued bipolar vague second category.

Definition 2.21: [10] An Interval-Valued bipolar vague set M in an Interval-Valued bipolar vague topological space (H, IBV_τ) is called an Interval-Valued bipolar vague H_δ -sets in (H, IBV_τ) if $M = \cup_{i=1}^\infty M_i$ where $M_i \in IBV_\tau$ for $i \in I$.

Definition 2.22: [10] An Interval-Valued bipolar vague set M in an Interval-Valued bipolar vague topological space (H, IBV_τ) is called an Interval-Valued bipolar vague J_σ -sets in (H, IBV_τ) if $M = \cup_{i=1}^\infty M_i$ where $M_i^c \in IBV_\tau$ for $i \in I$.

Definition 2.23: [10] An Interval-Valued bipolar vague topological space (H, IBV_τ) is called an Interval-Valued bipolar vague Volterra space if $IBVC(\cap_{i=1}^N M_i) = 1_\sim$, where M_i 's are Interval-Valued bipolar vague dense and Interval-Valued bipolar vague H_δ -sets in (H, IBV_τ) .

Definition 2.24: [10] Let M be a bipolar vague first category set in an Interval-Valued bipolar vague topological space (H, IBV_τ) . Then M^c is called an Interval-Valued bipolar vague residual sets in (H, IBV_τ) .

Definition 2.25: [10] An Interval-Valued bipolar vague topological space (H, IBV_{τ}) is called an Interval-Valued bipolar vague ε_{τ} -Volterra space if $IBVCl(\cap_{i=1}^N M_i) = 1_{\sim}$, where M_i 's are Interval-Valued bipolar vague dense and an Interval-Valued bipolar vague residual sets in (H, IBV_{τ}) .

Definition 2.26: [10] Let (H, IBV_{τ}) be an Interval-Valued bipolar vague topological space. Then (H, IBV_{τ}) is called an Interval-Valued bipolar vague Baire space if $IBVInt(\cup_{i=1}^{\infty} M_i) = 0_{\sim}$ where M_i 's are Interval-Valued bipolar vague nowhere dense sets in (H, IBV_{τ}) .

Definition 2.27: [10] Let (H, IBV_{τ}) be an Interval-Valued bipolar vague topological space. An Interval-Valued bipolar vague set M in (H, IBV_{τ}) is called an Interval-Valued bipolar vague σ -nowhere dense set if H is an Interval-Valued bipolar vague J_{σ} set in (H, IBV_{τ}) such that $IBVInt(M) = 0_{\sim}$.

Definition 2.28: [10] Let (H, IBV_{τ}) be an Interval-Valued bipolar vague topological space. Then (H, IBV_{τ}) is called an Interval-Valued bipolar vague σ -Baire space if $IBVInt(\cup_{i=1}^{\infty} M_i) = 0_{\sim}$, where M_i 's are Interval-Valued bipolar vague σ -nowhere dense sets.

Definition 2.29: [10] An Interval-Valued bipolar vague topological space (H, IBV_{τ}) is called an Interval-Valued bipolar vague weakly Volterra space if $IBVCl(\cap_{i=1}^N M_i) \neq 0_{\sim}$, where M_i 's are Interval-Valued bipolar vague dense and an Interval-Valued bipolar vague H_{δ} -sets in (H, IBV_{τ}) .

Definition 2.30: [10] Let (H, IBV_{τ}) be an Interval-Valued bipolar vague topological space. An Interval-Valued bipolar vague set M in (H, IBV_{τ}) is called an Interval-Valued bipolar vague σ -first category if $M = \cup_{i=1}^{\infty} M_i$, where M_i 's are Interval-Valued bipolar vague σ -nowhere dense set in (H, IBV_{τ}) . Any other Interval-Valued bipolar vague set in (H, IBV_{τ}) is said to be an Interval-Valued bipolar vague σ -second category in (H, IBV_{τ}) .

Definition 2.31: [10] An Interval-Valued bipolar vague topological space (H, IBV_{τ}) is an Interval-Valued bipolar vague σ -first category space if $1_{\sim} = \cup_{i=1}^{\infty} M_i$, where M_i 's are Interval-Valued bipolar vague σ -nowhere dense set in (H, IBV_{τ}) . (H, IBV_{τ}) is called an Interval-Valued bipolar vague σ -second category space if it is not an Interval-Valued bipolar vague σ -first category space.

Definition 2.32: [10] An Interval-Valued bipolar vague topological space (H, IBV_{τ}) is an Interval-Valued bipolar vague almost resolvable space if $\cup_{i=1}^{\infty} M_i = 1_{\sim}$, where the Interval-Valued bipolar vague sets M_i 's in (H, IBV_{τ}) are such that $IBVInt(M_i) = 0_{\sim}$. Otherwise, (H, IBV_{τ}) is called an Interval-Valued bipolar vague almost irresolvable.

Definition 2.33: [10] An Interval-Valued bipolar vague topological space (H, IBV_{τ}) is called an Interval-Valued bipolar vague \mathcal{P} -space if countable intersection of an Interval-Valued bipolar vague open sets in (H, IBV_{τ}) is an Interval-Valued bipolar vague open in (H, IBV_{τ}) .

Definition 2.34: [10] An Interval-Valued bipolar vague topological space (H, IBV_{τ}) is called an Interval-Valued bipolar vague submaximal space if for each an Interval-Valued bipolar vague set M in (H, IBV_{τ}) such that $IBVCl(M) = 1_{\sim}$, then $M \in \tau$.

III. INTERVAL-VALUED BIPOLAR VAGUE RESOLVABLE AND IRRESOLVABLE SPACES

Definition 3.1: Let (S, IBV_{τ}) be an Interval-Valued bipolar vague topological space. The (S, IBV_{τ}) is called an Interval-Valued bipolar vague resolvable if there exists an Interval-Valued bipolar vague dense set in (S, IBV_{τ}) such that $IBVCl(N^c) = 1_{\sim}$. Otherwise (S, IBV_{τ}) is called an Interval-Valued bipolar vague irresolvable.

Example 3.2: Let $S = \{p, q\}$. Define an Interval-Valued bipolar vague sets N , M and O as follows, $N = \langle s, [[0.3, 0.4][0.7, 0.8], [-0.7, -0.8][0.3, 0.4]], [[0.4, 0.5][0.8, 0.9], [-0.8, -0.9][0.4, 0.5]] \rangle$
 $M = \langle s, [[0.1, 0.2][0.5, 0.6], [-0.5, -0.6][0.1, 0.2]], [[0.2, 0.3][0.4, 0.5], [-0.4, -0.5][0.2, 0.3]] \rangle$ and $O = \langle s, [[0.2, 0.3][0.6, 0.7], [-0.6, -0.7][0.2, 0.3]], [[0.3, 0.5][0.5, 0.7], [-0.5, -0.7][0.3, 0.5]] \rangle$. Clearly $\tau = \{0_{\sim}, N, M, O, 1_{\sim}\}$ is an Interval-Valued bipolar vague topology in (S, IBV_{τ}) . Now $IBVInt(M) = 0_{\sim}$, $IBVInt(O) = 0_{\sim}$, $IBVInt(M^c) = 0_{\sim}$, $IBVInt(O^c) = 0_{\sim}$, $IBVCl(M) = 1_{\sim}$, $IBVCl(O) = 1_{\sim}$, $IBVCl(M^c) = 1_{\sim}$ and $IBVCl(O^c) = 1_{\sim}$. Since $IBVCl(M) = 1_{\sim}$ there exists an

Interval-Valued bipolar vague dense set M in (S, IBV_τ) . Therefore, the Interval-Valued bipolar vague topological space (S, IBV_τ) is called an Interval-Valued bipolar vague resolvable.

Example 3.3: Let $S = \{p, q\}$. Define an Interval-Valued bipolar vague sets N , M and O as follows, $N = \langle s, [[0.2, 0.3][0.5, 0.6], [-0.5, -0.6] [-0.2, -0.3]], [[0.1, 0.2][0.6, 0.7], [-0.6, -0.7] [-0.1, -0.2]] \rangle$
 $M = \langle s, [[0.3, 0.4][0.8, 0.9], [-0.8, -0.9] [-0.3, -0.4]], [[0.2, 0.3] [0.7, 0.8], [-0.7, -0.8] [-0.2, -0.3]] \rangle$ and $O = \langle s, [[0.4, 0.5][0.7, 0.8], [-0.7, -0.8] [-0.4, -0.5]], [[0.4, 0.5] [0.8, 0.9], [-0.8, -0.9] [-0.4, -0.5]] \rangle$. Clearly $\tau = \{0_\sim, N, M, O, 1_\sim\}$ is an Interval-Valued bipolar vague topology in (S, IBV_τ) . Now $IBVInt(M) = N$, $IBVInt(O) = N$, $IBVCl(M) = 1_\sim$, $IBVCl(O) = 1_\sim$. Therefore, M and O are Interval-Valued bipolar vague dense sets in (S, IBV_τ) . Then we have $IBVCl(M^c) = N^c$ and $IBVCl(O^c) = N^c$. Hence the Interval-Valued bipolar vague topological space (S, IBV_τ) is called an Interval-Valued bipolar vague irresolvable.

Proposition 3.4: Let (S, IBV_τ) be an Interval-Valued bipolar vague topological space. The (S, IBV_τ) is an Interval-Valued bipolar vague resolvable space if and only if (S, IBV_τ) has a pair of Interval-Valued bipolar vague dense set N_1 and N_2 such that $N_1 \subseteq N_2^c$.

Proof: Let (S, IBV_τ) be an Interval-Valued bipolar vague topological space. (S, IBV_τ) is an Interval-Valued bipolar vague resolvable space. Suppose that for all Interval-Valued bipolar vague dense set N_i and N_j we have $N_i \not\subseteq N_j^c$. Then $N_i \supset N_j^c$. Then $IBVCl(N_i) \supset IBVCl(N_j^c)$ which implies that $1_\sim \supset IBVCl(N_j^c)$ then $IBVCl(N_j^c) = 1_\sim$. Also $N_j \supset N_i^c$ then $IBVCl(N_j) \supset IBVCl(N_i^c)$ which implies that $1_\sim \supset IBVCl(N_i^c)$. Then $IBVCl(N_i^c) = 1_\sim$. Hence $IBVCl(N_i) = 1_\sim$, but $IBVCl(N_i^c) \neq 1_\sim$ for all Interval-Valued bipolar vague set N_i in (S, IBV_τ) which is a contradiction. Hence (S, IBV_τ) has a pair of an Interval-Valued bipolar vague dense set N_1 and N_2 such that $N_1 \subseteq N_2^c$.

Conversely, suppose that the Interval-Valued bipolar vague topological space (S, IBV_τ) has a pair of Interval-Valued bipolar vague dense set N_1 and N_2 such that $N_1 \subseteq N_2^c$. Suppose that (S, IBV_τ) is an Interval-Valued bipolar vague irresolvable space. Then for all Interval-Valued bipolar vague dense sets N_1 and N_2 in (S, IBV_τ) we have $IBVCl(N_1^c) \neq 1_\sim$. Then $IBVCl(N_2^c) \neq 1_\sim$ implies that there exists an Interval-Valued bipolar vague closed set M in (S, IBV_τ) such that $N_2^c \subset M \subset 1_\sim$. Then $N_1 \subseteq N_2^c \subset M \subset 1_\sim$ implies that $N_1 \subset M \subset 1_\sim$. Which is a contradiction. Hence (S, IBV_τ) is an Interval-Valued bipolar vague resolvable space.

Proposition 3.5: If (S, IBV_τ) is an Interval-Valued bipolar vague irresolvable if and only if $IBVInt(N) \neq 0_\sim$ for all Interval-Valued bipolar vague dense set A in (S, IBV_τ) .

Proof: Since (S, IBV_τ) is an Interval-Valued bipolar vague irresolvable space, for all Interval-Valued bipolar vague dense set N in (S, IBV_τ) , $IBVCl(N^c) \neq 1_\sim$. Then $(IBVInt(N))^c \neq 1_\sim$, which implies $IBVInt(N) \neq 0_\sim$.

Conversely, $IBVInt(N) \neq 0_\sim$, for all Interval-Valued bipolar vague dense set N in (S, IBV_τ) . Suppose that (S, IBV_τ) is an Interval-Valued bipolar vague resolvable. Then there exists an Interval-Valued bipolar vague dense set N in (S, IBV_τ) , such that $IBVCl(N^c) = 1_\sim$ implies that $(IBVInt(N))^c = 1_\sim$, implies $IBVInt(N) = 0_\sim$, which is a contradiction. Hence (S, IBV_τ) is an Interval-Valued bipolar vague irresolvable space.

Definition 3.6: An Interval-Valued bipolar vague topological space (S, IBV_τ) is called an Interval-Valued bipolar vague almost $G\mathcal{P}$ -space if $IBVInt(N) \neq 0_\sim$, for each non-zero Interval-Valued bipolar vague dense and an Interval-Valued bipolar vague G_δ -set N in (S, IBV_τ) . That is, (S, IBV_τ) is an Interval-Valued bipolar vague G_δ -set N in (S, IBV_τ) with $IBVCl(N) = 1_\sim$, $IBVInt(N) \neq 0_\sim$.

Proposition 3.7: If the Interval-Valued bipolar vague topological space (S, IBV_τ) is an Interval-Valued bipolar vague irresolvable space, then (S, IBV_τ) is an Interval-Valued bipolar vague almost $G\mathcal{P}$ -space.

Proof: Let N be an Interval-Valued bipolar vague dense and an Interval-Valued bipolar vague G_δ -set is an Interval-Valued bipolar vague irresolvable space (S, IBV_τ) . Since (S, IBV_τ) is an Interval-Valued bipolar vague irresolvable space, for the fuzzy dense set N in (S, IBV_τ) , we have $IBVCl(N^c) \neq 1_\sim$. But $IBVCl(N^c) = (IBVInt(N))^c \neq 1_\sim$, implies that $IBVInt(N) \neq 0_\sim$ and hence (S, IBV_τ) is an Interval-Valued bipolar vague almost $G\mathcal{P}$ -space.

Proposition 3.8: If the space (S, IBV_τ) is an Interval-Valued bipolar vague submaximal, then it is an Interval-Valued bipolar vague irresolvable.

Proof: Let (S, IBV_τ) be an Interval-Valued bipolar vague submaximal space. Assume that (S, IBV_τ) is an Interval-Valued bipolar vague resolvable space. Let N be an Interval-Valued bipolar vague dense set in (S, IBV_τ) . Then $IBVCl(N^c) \neq 1_\sim$. Hence $(IBVInt(N))^c \neq 1_\sim$, which implies that $IBVInt(N) = 0_\sim$. Then $N \notin \tau$, which is a contradiction to Interval-Valued bipolar vague submaximal space of (S, IBV_τ) . Hence (S, IBV_τ) is an Interval-Valued bipolar vague irresolvable space.

Proposition 3.9: If (S, IBV_τ) is an Interval-Valued bipolar vague Baire space and an Interval-Valued bipolar vague irresolvable space, then $IBVCl(\bigcup_{i=1}^\infty N_i) \neq 1_\sim$, where N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_τ) .

Proof: Let N be an Interval-Valued bipolar vague first category set in (S, IBV_τ) . Then $N = \bigcup_{i=1}^\infty N_i$, where N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_τ) . Since (S, IBV_τ) is an Interval-Valued bipolar vague Baire space, $IBVInt(N) = 0_\sim$. Then $(IBVInt(N))^c = 1_\sim$, which implies that $IBVCl(N^c) = 1_\sim$. Since (S, IBV_τ) is an Interval-Valued bipolar vague irresolvable space, $IBVCl((N^c)^c) \neq 1_\sim$. Hence $IBVCl(N) \neq 1_\sim$ and therefore $IBVCl(\bigcup_{i=1}^\infty N_i) \neq 1_\sim$, where N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_τ) .

Definition 3.10: An Interval-Valued bipolar vague topological space (S, IBV_τ) is said to be an Interval-Valued bipolar vague open hereditarily irresolvable if $IBVInt(IBVCl(N)) \neq 0_\sim$ then $IBVInt(N) \neq 0_\sim$ for any Interval-Valued bipolar vague set N in (S, IBV_τ) .

Example 3.11: Let $S = \{p, q\}$. Define an Interval-Valued bipolar vague sets N, M and O as follows, $N = \langle s, [[0.1, 0.2][0.5, 0.6], [-0.5, -0.6] [-0.1, -0.2]], [[0.1, 0.2][0.7, 0.8], [-0.7, -0.8] [-0.1, -0.2]] \rangle$ $M = \langle s, [[0.4, 0.5][0.7, 0.8], [-0.7, -0.8] [-0.4, -0.5]], [[0.2, 0.3] [0.8, 0.9], [-0.8, -0.9] [-0.2, -0.3]] \rangle$ and $O = \langle s, [[0.3, 0.4][0.6, 0.7], [-0.6, -0.7] [-0.3, -0.4]], [[0.2, 0.3][0.6, 0.7], [-0.5, -0.7] [-0.3, -0.5]] \rangle$. Clearly $\tau = \{0_\sim, N, M, O, 1_\sim\}$ is an Interval-Valued bipolar vague topology in (S, IBV_τ) . Now $IBVCl(N) = N^c$, $IBVCl(M) = 1_\sim$ and $IBVCl(O) = N^c$. Also $IBVInt(IBVCl(N)) = IBVInt(N^c) = N \neq 0_\sim$ and $IBVInt(N^c) = N \neq 0_\sim$, $IBVInt(IBVCl(N)) = IBVInt(1_\sim) = 1_\sim \neq 0_\sim$ and $IBVInt(M) = M \neq 0_\sim$, $IBVInt(IBVCl(O)) = N \neq 0_\sim$ and $IBVInt(O) = N \neq 0_\sim$, $IBVInt(IBVCl(N^c)) = N \neq 0_\sim$, $IBVInt(IBVCl(M^c)) = N \neq 0_\sim$ then $IBVInt(N) \neq 0_\sim$ for any nonzero Interval-Valued bipolar vague set N in (S, IBV_τ) . Therefore (S, IBV_τ) is an Interval-Valued bipolar vague open hereditarily irresolvable space.

Proposition 3.12: Let (S, IBV_τ) be an Interval-Valued bipolar vague topological space. If (S, IBV_τ) is an Interval-Valued bipolar vague open hereditarily irresolvable then (S, IBV_τ) is an Interval-Valued bipolar vague irresolvable.

Proof: Let N be an Interval-Valued bipolar vague dense set in (S, IBV_τ) . Then $IBVCl(N) = 1_\sim$, which implies that $IBVInt(IBVCl(N)) = 1_\sim \neq 0_\sim$. Since (S, IBV_τ) is an Interval-Valued bipolar vague open hereditarily irresolvable, we have $IBVInt(N) \neq 0_\sim$. Therefore, by Proposition 3.5, $IBVInt(N) \neq 0_\sim$ for all Interval-Valued bipolar vague dense set in (S, IBV_τ) , implies that (S, IBV_τ) is an Interval-Valued bipolar vague irresolvable. The converse is not true is shown by the example.

Example 3.13: Let $S = \{p, q\}$. Define an Interval-Valued bipolar vague sets N, M and O as follows, $N = \langle s, [[0.2, 0.3][0.5, 0.6], [-0.5, -0.6] [-0.2, -0.3]], [[0.3, 0.4][0.6, 0.7], [-0.6, -0.7] [-0.3, -0.4]] \rangle$ $M = \langle s, [[0.3, 0.4][0.6, 0.7], [-0.6, -0.7] [-0.3, -0.4]], [[0.4, 0.5] [0.7, 0.8], [-0.7, -0.8] [-0.4, -0.5]] \rangle$ and $O = \langle s, [[0.3, 0.4][0.7, 0.8], [-0.7, -0.8] [-0.3, -0.4]], [[0.4, 0.5][0.6, 0.7], [-0.6, -0.7] [-0.4, -0.5]] \rangle$. Clearly $\tau = \{0_\sim, N, M, O, 1_\sim\}$ is an Interval-Valued bipolar vague topology in (S, IBV_τ) . Now O and 1_\sim are Interval-Valued bipolar vague dense set in (S, IBV_τ) . Then $IBVInt(O) = N \neq 0_\sim$, $IBVInt(1_\sim) \neq 0_\sim$. Then we have $IBVCl(O^c) = N^c = N \neq 0_\sim$ and $IBVInt(O^c) = 0_\sim$. Thus (S, IBV_τ) is not an Interval-Valued bipolar vague open hereditarily irresolvable space.

Proposition 3.14: Let (S, IBV_τ) be an Interval-Valued bipolar vague open hereditarily irresolvable. Then $IBVInt(N) \not\subset (IBVInt(M))^c$ for any two Interval-Valued bipolar vague dense sets N and M in (S, IBV_τ) .

Proof: Let N and M be any two Interval-Valued bipolar vague dense sets in (S, IBV_τ) . Then $IBVCl(N) = 1_\sim$ and $IBVCl(M) = 1_\sim$ implies that $IBVInt(IBVCl(N)) \neq 0_\sim$ and $IBVInt(IBVCl(M)) \neq 0_\sim$. Since (S, IBV_τ) is an Interval-

Valued bipolar vague open hereditarily irresolvable, $IBVInt(N) \neq 0_{\sim}$ and $IBVInt(M) \neq 0_{\sim}$. Hence by Proposition 3.4, $N \not\subset M^c$. Therefore $IBVInt(N) \subset N \not\subset M^c \subset (IBVInt(M))^c$. Hence we have $IBVInt(N) \not\subset (IBVInt(M))^c$ for any two Interval-Valued bipolar vague dense sets N and M in (S, IBV_{τ}) .

Proposition 3.15: Let (S, IBV_{τ}) be an Interval-Valued bipolar vague topological space. If (S, IBV_{τ}) is an Interval-Valued bipolar vague open hereditarily irresolvable then $IBVInt(N) = 0_{\sim}$ for any non-zero Interval-Valued bipolar vague dense set N in (S, IBV_{τ}) implies that $IBVInt(IBVCl(N)) = 0_{\sim}$.

Proof: Let A be an Interval-Valued bipolar vague set in (S, IBV_{τ}) , such that $IBVInt(N) = 0_{\sim}$. We claim that $IBVInt(IBVCl(N)) = 0_{\sim}$. Suppose that $IBVInt(IBVCl(N)) \neq 0_{\sim}$. Since (S, IBV_{τ}) is an Interval-Valued bipolar vague open hereditarily irresolvable, we have $IBVInt(N) \neq 0_{\sim}$, which is a contradiction to $IBVInt(N) = 0_{\sim}$. Hence $IBVInt(IBVCl(N)) = 0_{\sim}$.

Proposition 3.16: If the Interval-Valued bipolar vague topological space (S, IBV_{τ}) is an Interval-Valued bipolar vague open hereditarily irresolvable and Interval-Valued bipolar vague second category space, then (S, IBV_{τ}) is an Interval-Valued bipolar vague weakly Volterra space.

Proof: Assume that (S, IBV_{τ}) is not an Interval-Valued bipolar vague weakly Volterra space. Therefore, $IBVCl(\bigcap_{i=1}^N N_i) = 0_{\sim}$, where N_i 's are Interval-Valued bipolar vague dense and Interval-Valued bipolar vague G_{δ} -set in (S, IBV_{τ}) . Then $IBVCl(\bigcap_{i=1}^N N_i) = 0_{\sim} \Rightarrow IBVInt(\bigcup_{i=1}^N (N_i^c)) = 1_{\sim} \Rightarrow \bigcup_{i=1}^N (N_i^c) = 1_{\sim}$. Now $\bigcup_{i=1}^N (N_i^c) \subseteq \bigcup_{i=1}^{\infty} (N_i^c) \Rightarrow \bigcup_{i=1}^{\infty} (N_i^c) = 1_{\sim}$, since $IBVCl(N_i) = 1_{\sim}$, we have $IBVInt(N_i^c) = 0_{\sim}$. Since (S, IBV_{τ}) is an Interval-Valued bipolar vague open hereditarily irresolvable, $IBVInt(N_i^c) = 0_{\sim}$, which implies that $IBVInt(IBVCl(N_i^c)) = 0_{\sim}$. Therefore N_i^c 's are Interval-Valued bipolar vague nowhere dense set. Therefore $\bigcup_{i=1}^{\infty} (N_i^c) = 1_{\sim}$, where N_i 's are Interval-Valued bipolar vague nowhere dense sets. This implies that (S, IBV_{τ}) is an Interval-Valued bipolar vague first category space, which is a contradiction since (S, IBV_{τ}) is an Interval-Valued bipolar vague second category space. Hence (S, IBV_{τ}) is an Interval-Valued bipolar vague weakly Volterra space.

Proposition 3.17: Let (S, IBV_{τ}) be an Interval-Valued bipolar vague topological space. If (S, IBV_{τ}) is an Interval-Valued bipolar vague open hereditarily irresolvable then $IBVCl(N) = 1_{\sim}$ for any non-zero Interval-Valued bipolar vague dense set N in (S, IBV_{τ}) implies that $IBVCl(IBVInt(N)) = 1_{\sim}$.

Proof: Let N be an Interval-Valued bipolar vague set in (S, IBV_{τ}) , such that $IBVCl(N) = 1_{\sim}$. Then we have $(IBVCl(N))^c = 0_{\sim}$, which implies that $IBVInt(N^c) = 0_{\sim}$. Since (S, IBV_{τ}) is an Interval-Valued open hereditarily irresolvable by proposition 3.15, we have $IBVInt(IBVCl(N^c)) = 0_{\sim}$. Therefore $(IBVCl(IBVInt(N)))^c = 0_{\sim}$ implies that $IBVCl(IBVInt(N)) = 1_{\sim}$.

Proposition 3.18: If $IBVCl(\bigcap_{i=1}^{\infty} N_i) = 1_{\sim}$, where N_i 's are Interval-Valued bipolar vague dense set in an Interval-Valued bipolar vague open hereditarily irresolvable space, then (S, IBV_{τ}) is an Interval-Valued bipolar vague Baire space.

Proof: Now $IBVCl(\bigcap_{i=1}^{\infty} N_i) = 1_{\sim}$, where $IBVCl(N_i) = 1_{\sim}$, implies that $IBVInt(\bigcup_{i=1}^{\infty} N_i^c) = 0_{\sim}$, where $IBVInt(N_i^c) = 0_{\sim}$. Let $M_i = N_i^c$. Then $IBVInt(\bigcup_{i=1}^{\infty} M_i) = 0_{\sim}$, where $IBVInt(M_i) = 0_{\sim}$. Since (S, IBV_{τ}) is an Interval-Valued bipolar vague open hereditarily irresolvable space, $IBVInt(M_i) = 0_{\sim}$ implies that $IBVInt(IBVCl(M_i)) = 0_{\sim}$. Hence M_i is an Interval-Valued bipolar vague nowhere dense set in (S, IBV_{τ}) . Hence $IBVInt(\bigcup_{i=1}^{\infty} M_i) = 0_{\sim}$, where M_i 's are Interval-Valued bipolar vague nowhere dense set in (S, IBV_{τ}) , implies that (S, IBV_{τ}) is an Interval-Valued bipolar vague Baire space.

Proposition 3.19: If the Interval-Valued bipolar vague topological space (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -Baire space and Interval-Valued bipolar vague open hereditarily irresolvable, then (S, IBV_{τ}) is an Interval-Valued bipolar vague Baire space.

Proof: Let (S, IBV_{τ}) be an Interval-Valued bipolar vague σ -Baire space and an Interval-Valued bipolar vague open hereditarily irresolvable space. Then $IBVInt(\bigcup_{i=1}^{\infty} N_i) = 0_{\sim}$ where N_i 's are Interval-Valued bipolar vague σ -nowhere dense set in (S, IBV_{τ}) . Then N_i^c 's are Interval-Valued bipolar vague dense set in (S, IBV_{τ}) . Hence, $IBVInt(\bigcup_{i=1}^{\infty} N_i) = 0_{\sim}$

0_{\sim} , where N_i^c 's are Interval-Valued bipolar vague dense set in (S, IBV_{τ}) . Therefore (S, IBV_{τ}) is an Interval-Valued bipolar vague Baire space.

IV.. INTERVAL-VALUED BIPOLAR VAGUE ALMOST RESOLVABLE AND ALMOST IRRESOLVABLE SPACES

Proposition 4.1: If $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague dense sets in an Interval-Valued bipolar vague topological space (S, IBV_{τ}) , then (S, IBV_{τ}) is an Interval-Valued bipolar vague almost resolvable space.

Proof: Suppose that $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$, where $IBVCl(N_i) = 1_{\sim}$ in (S, IBV_{τ}) . Then we have $(\bigcap_{i=1}^{\infty} N_i)^c = (0_{\sim})^c = 1_{\sim}$, where $(IBVCl(N_i))^c = 0_{\sim}$. This implies that $\bigcup_{i=1}^{\infty} M_i = 1_{\sim}$, where $IBVInt(N_i^c) = 0_{\sim}$. Let $N_i^c = M_i$. Then we have $\bigcup_{i=1}^{\infty} N_i^c = 1_{\sim}$, where $IBVInt(M_i) = 0_{\sim}$, in (S, IBV_{τ}) . Hence (S, IBV_{τ}) is an Interval-Valued bipolar vague almost resolvable space.

Proposition 4.2: If each Interval-Valued bipolar vague set N_i is an Interval-Valued bipolar vague F_{σ} -set in an Interval-Valued bipolar vague almost resolvable space (S, IBV_{τ}) , then $\bigcap_{i=1}^{\infty} N_i^c = 0_{\sim}$, where N_i^c 's are Interval-Valued bipolar vague dense sets in (S, IBV_{τ}) .

Proof: Let (S, IBV_{τ}) be an Interval-Valued bipolar vague almost resolvable space. Then $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where the Interval-Valued bipolar vague sets N_i 's in (S, IBV_{τ}) are such that $IBVInt(N_i) = 0_{\sim}$. This implies that $\bigcap_{i=1}^{\infty} N_i^c = 0_{\sim}$ and $IBVCl(N_i^c) = 1_{\sim}$. Since the Interval-Valued bipolar vague sets N_i 's in (S, IBV_{τ}) are Interval-Valued bipolar vague F_{σ} -sets, N_i^c 's are Interval-Valued bipolar vague G_{δ} -sets in (S, IBV_{τ}) . Hence, we have $\bigcap_{i=1}^{\infty} N_i^c = 0_{\sim}$, where N_i^c 's are Interval-Valued bipolar vague dense sets in (S, IBV_{τ}) .

Proposition 4.3: If $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague residual sets in an Interval-Valued bipolar vague Baire space (S, IBV_{τ}) , then (S, IBV_{τ}) is an Interval-Valued bipolar vague almost resolvable space.

Proof: Let (S, IBV_{τ}) be an Interval-Valued bipolar vague Baire space in which $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague residual sets in (S, IBV_{τ}) . Then by Proposition, $IBVCl(N_i) = 1_{\sim}$. Then $(IBVCl(N_i))^c = 0_{\sim}$ and hence $IBVInt(N_i^c) = 0_{\sim}$. Now $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$, implies that $\bigcup_{i=1}^{\infty} N_i^c = 1_{\sim}$. Let $N_i^c = M_i$. Hence $\bigcup_{i=1}^{\infty} M_i = 1_{\sim}$, where $IBVInt(M_i) = 0_{\sim}$. Therefore (S, IBV_{τ}) is an Interval-Valued bipolar vague almost resolvable space.

Proposition 4.4: If the Interval-Valued bipolar vague topological space (S, IBV_{τ}) is an Interval-Valued bipolar vague Baire space, then (S, IBV_{τ}) is an Interval-Valued bipolar vague almost irresolvable space.

Proof: Suppose (S, IBV_{τ}) is an Interval-Valued bipolar vague Baire space then, $IBVInt(\bigcup_{i=1}^{\infty} N_i) = 0_{\sim}$, where N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_{τ}) . This implies that $IBVInt(IBVCl(N_i)) = 0_{\sim}$. Suppose that (S, IBV_{τ}) is an Interval-Valued bipolar vague almost resolvable space. Then $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where $IBVInt(N_i) = 0_{\sim}$. Now $IBVInt(\bigcup_{i=1}^{\infty} N_i) = IBVInt(1_{\sim}) = 1_{\sim}$, which is that $0_{\sim} = 1_{\sim}$, a contradiction. Hence, we must have $\bigcup_{i=1}^{\infty} N_i \neq 1_{\sim}$, where $IBVInt(N_i) = 0_{\sim}$. Therefore (S, IBV_{τ}) is an Interval-Valued bipolar vague almost irresolvable space.

Proposition 4.5: If $IBVCl(\bigcap_{i=1}^{\infty} N_i) = 1_{\sim}$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague dense and Interval-Valued bipolar vague G_{δ} -sets in an Interval-Valued bipolar vague submaximal space in (S, IBV_{τ}) , then (S, IBV_{τ}) is an Interval-Valued bipolar vague almost irresolvable space.

Proof: Suppose that $IBVCl(\bigcap_{i=1}^{\infty} N_i) = 1_{\sim}$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague dense and Interval-Valued bipolar vague G_{δ} -sets in an Interval-Valued bipolar vague submaximal space in (S, IBV_{τ}) . Now $IBVCl(\bigcap_{i=1}^{\infty} N_i) = 1_{\sim}$ implies that $IBVInt(\bigcup_{i=1}^{\infty} (N_i^c)) = 0_{\sim}$. Since N_i 's are Interval-Valued bipolar vague dense in an Interval-Valued bipolar vague submaximal space, N_i 's are Interval-Valued bipolar vague open sets in (S, IBV_{τ}) and hence N_i^c 's are Interval-Valued bipolar vague closed sets in (S, IBV_{τ}) . Then $IBVCl(N_i^c) = N_i^c$. Since N_i 's are Interval-Valued bipolar vague dense in (S, IBV_{τ}) , $IBVCl(N_i) = 1_{\sim}$. Then $(IBVCl(N_i))^c = 0_{\sim}$ this implies that

$IBVInt(N_i^c) = 0_{\sim}$. Now $IBVInt(IBVCl(N_i^c)) = IBVInt(N_i^c)$ implies that $IBVInt(IBVCl(N_i^c)) = 0_{\sim}$. Then N_i^c 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_{τ}) . Hence we have $IBVInt(\bigcup_{i=1}^{\infty} (N_i^c)) = 0_{\sim}$ where N_i^c 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_{τ}) . Hence (S, IBV_{τ}) is an Interval-Valued bipolar vague Baire space and so by Proposition 4.4, (S, IBV_{τ}) is an Interval-Valued bipolar vague almost irresolvable space.

Proposition 4.6: If each Interval-Valued bipolar vague set is an Interval-Valued bipolar vague F_{σ} -set in an Interval-Valued bipolar vague almost resolvable space (S, IBV_{τ}) , then (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -first category space.

Proof: Let (S, IBV_{τ}) be an Interval-Valued bipolar vague almost resolvable space such that each Interval-Valued bipolar vague set in (S, IBV_{τ}) is an Interval-Valued bipolar vague F_{σ} -set. Then we have, $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where the Interval-Valued bipolar vague sets N_i 's in (S, IBV_{τ}) are such that $IBVInt(N_i) = 0_{\sim}$. Hence the Interval-Valued bipolar vague sets N_i 's in (S, IBV_{τ}) are Interval-Valued bipolar vague F_{σ} -sets such that $IBVInt(N_i) = 0_{\sim}$. This implies that N_i 's are Interval-Valued bipolar vague σ -nowhere dense sets in (S, IBV_{τ}) . Thus, we have $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where the N_i 's are Interval-Valued bipolar vague σ -nowhere dense sets in (S, IBV_{τ}) . Therefore (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -first category space.

Proposition 4.7: If $IBVCl(IBVInt(N)) = 1_{\sim}$, for each Interval-Valued bipolar vague dense set N in an Interval-Valued bipolar vague almost resolvable space (S, IBV_{τ}) , then (S, IBV_{τ}) is an Interval-Valued bipolar vague first category space.

Proof: Let (S, IBV_{τ}) be an Interval-Valued bipolar vague almost resolvable space such that $IBVCl(IBVInt(N)) = 1_{\sim}$, for each Interval-Valued bipolar vague dense set N in (S, IBV_{τ}) . Since (S, IBV_{τ}) is an Interval-Valued bipolar vague almost resolvable, $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where the Interval-Valued bipolar vague sets N_i 's in (S, IBV_{τ}) are such that $IBVInt(N_i) = 0_{\sim}$. Now $(IBVInt(N_i))^c = 1_{\sim}$, implies that $IBVCl(N_i^c) = 1_{\sim}$. Then by hypothesis, $IBVCl(IBVInt(N_i^c)) = 1_{\sim}$, for the Interval-Valued bipolar vague dense set N_i^c in (S, IBV_{τ}) . This implies that $IBVInt(IBVCl(N_i^c)) = 0_{\sim}$. Hence N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_{τ}) . Therefore $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_{τ}) , implies that (S, IBV_{τ}) is an Interval-Valued bipolar vague first category space.

Proposition 4.8: If the Interval-Valued bipolar vague topological space (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -second category space, then (S, IBV_{τ}) is an Interval-Valued bipolar vague almost irresolvable space.

Proof: Let (S, IBV_{τ}) be an Interval-Valued bipolar vague σ -second category space. Then, $\bigcup_{i=1}^{\infty} N_i \neq 1_{\sim}$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague σ -nowhere dense sets in (S, IBV_{τ}) . Therefore N_i 's are Interval-Valued bipolar vague F_{σ} -sets in (S, IBV_{τ}) such that $IBVInt(N_i) = 0_{\sim}$. Hence, we have $\bigcup_{i=1}^{\infty} N_i \neq 1_{\sim}$, where $IBVInt(N_i) = 0_{\sim}$ implies (S, IBV_{τ}) is an Interval-Valued bipolar vague almost irresolvable space.

Proposition 4.9: If $IBVCl(IBVInt(N)) = 1_{\sim}$ for each Interval-Valued bipolar vague dense set in an Interval-Valued bipolar vague almost resolvable space (S, IBV_{τ}) , then (S, IBV_{τ}) is not an Interval-Valued bipolar vague Baire space.

Proof: Let (S, IBV_{τ}) be an Interval-Valued bipolar vague almost resolvable space such that $IBVCl(IBVInt(N)) = 1_{\sim}$, for each Interval-Valued bipolar vague dense set N in (S, IBV_{τ}) . Then by Proposition 4.7, (S, IBV_{τ}) is an Interval-Valued bipolar vague first category space. This implies that $IBVInt(\bigcup_{i=1}^{\infty} N_i) = IBVInt(1_{\sim}) = 1_{\sim} \neq 0_{\sim}$. Hence (S, IBV_{τ}) is not an Interval-Valued bipolar vague Baire space.

Proposition 4.10: If $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where the Interval-Valued bipolar vague sets N_i 's are non-zero Interval-Valued bipolar vague open sets in an Interval-Valued bipolar vague topological space (S, IBV_{τ}) , then (S, IBV_{τ}) is an Interval-Valued bipolar vague almost irresolvable space.

Proof: Suppose that $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where the Interval-Valued bipolar vague sets N_i 's are non-zero Interval-Valued bipolar vague open sets in (S, IBV_{τ}) , $IBVInt(N_i) = N_i \neq 0_{\sim}$. Therefore (S, IBV_{τ}) is an Interval-Valued bipolar vague almost irresolvable space.

Definition 4.11: An Interval-Valued bipolar vague topological space (S, IBV_τ) is called an Interval-Valued bipolar vague hyper connected space if every Interval-Valued bipolar vague open set is an Interval-Valued bipolar vague dense in (S, IBV_τ) .

Proposition 4.12: If $\bigcap_{i=1}^{\infty} N_i = 0_\sim$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague open sets in an Interval-Valued bipolar vague hyper connected space in (S, IBV_τ) , then (S, IBV_τ) is an Interval-Valued bipolar vague almost resolvable space.

Proof: Suppose that $\bigcap_{i=1}^{\infty} N_i = 0_\sim$ where $N_i \in IBV_\tau$ and N_i 's are Interval-Valued bipolar vague open sets in an Interval-Valued bipolar vague hyper connected space then N_i 's are Interval-Valued bipolar vague dense set in (S, IBV_τ) for each i . Then by proposition 4.1, (S, IBV_τ) is an Interval-Valued bipolar vague almost resolvable space.

Proposition 4.13: If $\bigcup_{i=1}^{\infty} M_i = 1_\sim$, where the Interval-Valued bipolar vague sets M_i 's are Interval-Valued bipolar vague σ -nowhere dense sets in an Interval-Valued bipolar vague topological space (S, IBV_τ) , then (S, IBV_τ) is an Interval-Valued bipolar vague almost resolvable space.

Proof: Let M_i 's be an Interval-Valued bipolar vague σ -nowhere dense sets in (S, IBV_τ) . Then M_i 's are Interval-Valued bipolar vague F_σ -sets with $IBVInt(M_i) = 0_\sim$. Now $\bigcup_{i=1}^{\infty} M_i = 1_\sim$, where $IBVInt(M_i) = 0_\sim$, implies that (S, IBV_τ) is an Interval-Valued bipolar vague almost resolvable space.

Proposition 4.14: If $\bigcap_{i=1}^{\infty} N_i = 0_\sim$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague G_δ -sets in an Interval-Valued bipolar vague hyper connected and Interval-Valued bipolar vague \mathcal{P} -space, then (S, IBV_τ) is an Interval-Valued bipolar vague almost resolvable space.

Proof: Let N_i 's be an Interval-Valued bipolar vague G_δ -sets in an Interval-Valued bipolar vague \mathcal{P} -space in (S, IBV_τ) . Then N_i 's are Interval-Valued bipolar vague open sets in (S, IBV_τ) . Hence, we have $\bigcap_{i=1}^{\infty} N_i = 0_\sim$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague open sets in an Interval-Valued bipolar vague hyper connected space (S, IBV_τ) . Therefore, by Proposition 4.12, (S, IBV_τ) is an Interval-Valued bipolar vague almost resolvable space.

Definition 4.15: An Interval-Valued bipolar vague topological space (S, IBV_τ) is called an Interval-Valued bipolar vague nodec space, if every non-zero Interval-Valued bipolar vague nowhere dense set in (S, IBV_τ) , is an Interval-Valued bipolar vague closed set in (S, IBV_τ) .

Proposition 4.16: If (S, IBV_τ) is an Interval-Valued bipolar vague first category space and Interval-Valued bipolar vague nodec space, then (S, IBV_τ) is an Interval-Valued bipolar vague almost resolvable space.

Proof: Let (S, IBV_τ) is an Interval-Valued bipolar vague first category space. Then $\bigcup_{i=1}^{\infty} N_i = 1_\sim$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_τ) . Since (S, IBV_τ) is an Interval-Valued bipolar vague nodec space, that is, $IBVCl(N_i) = N_i$. Now $IBVInt(IBVCl(N_i)) = 0_\sim$, implies that $IBVInt(N_i) = 0_\sim$. Hence (S, IBV_τ) is an Interval-Valued bipolar vague almost resolvable space.

Proposition 4.17: If the Interval-Valued bipolar vague topological space (S, IBV_τ) is an Interval-Valued bipolar vague second category space, then (S, IBV_τ) is an Interval-Valued bipolar vague almost irresolvable space.

Proof: Let (S, IBV_τ) be an Interval-Valued bipolar vague second category space. Then $\bigcup_{i=1}^{\infty} N_i \neq 1_\sim$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_τ) . That is $\bigcup_{i=1}^{\infty} N_i \neq 1_\sim$, where $IBVInt(IBVCl(N_i)) = 0_\sim$. Now $IBVInt(N_i) \subseteq IBVInt(IBVCl(N_i))$, implies that $IBVInt(N_i) = 0_\sim$. Hence $\bigcup_{i=1}^{\infty} N_i \neq 1_\sim$, where $IBVInt(N_i) = 0_\sim$ and therefore (S, IBV_τ) is an Interval-Valued bipolar vague almost irresolvable space.

Proposition 4.18: If the Interval-Valued bipolar vague almost resolvable space (S, IBV_τ) is an Interval-Valued bipolar vague submaximal space, then (S, IBV_τ) is an Interval-Valued bipolar vague first category space.

Proof: Let (S, IBV_τ) be an Interval-Valued bipolar vague almost resolvable space. Then $\bigcup_{i=1}^{\infty} N_i = 1_\sim$, where the Interval-Valued bipolar vague sets N_i 's in (S, IBV_τ) are such that $IBVInt(N_i) = 0_\sim$. Then we have $\bigcap_{i=1}^{\infty} N_i^c = 0_\sim$, where $IBVCl(N_i^c) = 1_\sim$. Since the space (S, IBV_τ) is an Interval-Valued bipolar vague submaximal space, the Interval-Valued bipolar vague dense sets N_i^c 's are Interval-Valued bipolar vague open sets in (S, IBV_τ) . Then N_i 's are Interval-Valued bipolar vague closed sets in (S, IBV_τ) and hence $IBVCl(N_i) = N_i$. Now $IBVInt(IBVCl(N_i)) = IBVInt(N_i) = 0_\sim$. Then N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_τ) . Hence $\bigcup_{i=1}^{\infty} N_i = 1_\sim$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_τ) implies that (S, IBV_τ) is an Interval-Valued bipolar vague first category space.

Proposition 4.19: If the Interval-Valued bipolar vague almost irresolvable space (S, IBV_τ) is an Interval-Valued bipolar vague submaximal space, then (S, IBV_τ) is an Interval-Valued bipolar vague second category space.

Proof: Let (S, IBV_τ) be an Interval-Valued bipolar vague almost irresolvable space. Then $\bigcup_{i=1}^{\infty} N_i \neq 1_\sim$, where the Interval-Valued bipolar vague sets N_i 's are such that $IBVInt(N_i) = 0_\sim$. Now $IBVInt(N_i) = 0_\sim$, implies that $IBVCl(N_i^c) = 1_\sim$. That is N_i^c 's are Interval-Valued bipolar vague dense sets in (S, IBV_τ) . Since (S, IBV_τ) is an Interval-Valued bipolar vague submaximal space, the Interval-Valued bipolar vague dense sets N_i^c 's are Interval-Valued bipolar vague open sets in (S, IBV_τ) . Then N_i 's are Interval-Valued bipolar vague closed sets in (S, IBV_τ) . That is, $IBVCl(N_i) = N_i$. Now $IBVInt(N_i) = 0_\sim$, implies that $IBVInt(IBVCl(N_i)) = 0_\sim$. Then N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_τ) . Hence, we have $\bigcup_{i=1}^{\infty} N_i \neq 1_\sim$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_τ) . Therefore (S, IBV_τ) is an Interval-Valued bipolar vague second category space.

Proposition 4.20: If the Interval-Valued bipolar vague almost resolvable space (S, IBV_τ) is an Interval-Valued bipolar vague submaximal space, then (S, IBV_τ) is not an Interval-Valued bipolar vague Baire space.

Proof: Let the Interval-Valued bipolar vague almost resolvable space (S, IBV_τ) be an Interval-Valued bipolar vague submaximal space. Then by proposition 4.18, (S, IBV_τ) is an Interval-Valued bipolar vague first category space and hence $\bigcup_{i=1}^{\infty} N_i = 1_\sim$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_τ) . Now $IBVInt(\bigcup_{i=1}^{\infty} N_i) = IBVInt(1_\sim) = 1_\sim \neq 0_\sim$. Hence (S, IBV_τ) is not an Interval-Valued bipolar vague Baire space.

Proposition 4.21: If the Interval-Valued bipolar vague almost resolvable space (S, IBV_τ) is an Interval-Valued bipolar vague open hereditarily irresolvable space, then (S, IBV_τ) is not an Interval-Valued bipolar vague Baire space.

Proof: Let (S, IBV_τ) be an Interval-Valued bipolar vague almost resolvable space. Then $\bigcup_{i=1}^{\infty} N_i = 1_\sim$, where the Interval-Valued bipolar vague sets N_i 's in (S, IBV_τ) are such that $IBVInt(N_i) = 0_\sim$. Since (S, IBV_τ) is an Interval-Valued bipolar vague open hereditarily irresolvable space, $IBVInt(N_i) = 0_\sim$ implies that $IBVInt(IBVCl(N_i)) = 0_\sim$. Now $IBVInt(\bigcup_{i=1}^{\infty} N_i) = IBVInt(1_\sim) \neq 0_\sim$. Hence (S, IBV_τ) is not an Interval-Valued bipolar vague Baire space.

Proposition 4.22: If the Interval-Valued bipolar vague almost irresolvable space (S, IBV_τ) is an Interval-Valued bipolar vague open hereditarily irresolvable space, then (S, IBV_τ) is an Interval-Valued bipolar vague second category space.

Proof: Let (S, IBV_τ) be an Interval-Valued bipolar vague almost irresolvable space. Then $\bigcup_{i=1}^{\infty} N_i \neq 1_\sim$, where the Interval-Valued bipolar vague sets N_i 's in (S, IBV_τ) are such that $IBVInt(N_i) = 0_\sim$. Since (S, IBV_τ) is an Interval-Valued bipolar vague open hereditarily irresolvable space, $IBVInt(N_i) = 0_\sim$ implies that $IBVInt(IBVCl(N_i)) = 0_\sim$. Then N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_τ) . Hence $\bigcup_{i=1}^{\infty} N_i \neq 1_\sim$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_τ) , implies that (S, IBV_τ) is an Interval-Valued bipolar vague second category space.

V. INTERVAL-VALUED BIPOLAR VAGUE σ -RESOLVABLE SPACES

Definition 5.1: An Interval-Valued bipolar vague topological space (S, IBV_{τ}) is called an Interval-Valued bipolar vague σ -resolvable space if $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where N_i 's are Interval-Valued bipolar vague dense set in (S, IBV_{τ}) such that $N_i \subseteq N_j^c$, for $i \neq j$. An Interval-Valued bipolar vague topological space, which is not an Interval-Valued bipolar vague σ -resolvable space, is an Interval-Valued bipolar vague σ -irresolvable space.

Proposition 5.2: If the Interval-Valued bipolar vague topological space (S, IBV_{τ}) is called an Interval-Valued bipolar vague σ -resolvable space then $IBVInt(N_i) = 0_{\sim}$.

Proof: Let (S, IBV_{τ}) be an Interval-Valued bipolar vague σ -resolvable space if $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where N_i 's are Interval-Valued bipolar vague dense set in (S, IBV_{τ}) such that $N_i \subseteq N_j^c$, for $i \neq j$. Then we have $IBVCl(N_j) \subseteq IBVCl(N_i^c)$. Since N_j 's are Interval-Valued bipolar vague dense set in (S, IBV_{τ}) , $IBVCl(N_j) = 1_{\sim}$. Hence $1_{\sim} \subseteq IBVCl(N_i^c)$. That is $IBVCl(N_i^c) = 1_{\sim}$. Then, we have $(IBVInt(N_i))^c = 1_{\sim}$. This implies that $IBVInt(N_i) = 0_{\sim}$. Also $N_j \subseteq N_i^c$ implies that $IBVInt(N_j) \subseteq IBVInt(N_i^c)$. Then $IBVInt(N_j) \subseteq (IBVCl(N_i))^c$. This implies that $IBVInt(N_j) \subseteq 0_{\sim}$. That is, $IBVInt(N_j) = 0_{\sim}$. Hence we have $IBVInt(N_i) = 0_{\sim}$ for all i .

Proposition 5.3: If the Interval-Valued bipolar vague topological space (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space, then (S, IBV_{τ}) is an Interval-Valued bipolar vague almost resolvable space.

Proof: Suppose that the Interval-Valued bipolar vague topological space (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space. Then by Proposition 5.2, we have $IBVInt(N_i) = 0_{\sim}$. Therefore (S, IBV_{τ}) is an Interval-Valued bipolar vague almost resolvable space.

Proposition 5.4: Let N_i 's be an Interval-Valued bipolar vague sets in an Interval-Valued bipolar vague topological space (S, IBV_{τ}) such that $IBVInt(N_i) = 0_{\sim}$ for all $i \in I$ and $N_i \supseteq N_j^c$, for $i \neq j$. If $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$ then (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space.

Proof: Suppose that $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$, where N_i 's are Interval-Valued bipolar vague set in (S, IBV_{τ}) such that $IBVInt(N_i) = 0_{\sim}$ for all $i \in I$ and $N_i \supseteq N_j^c$, for $i \neq j$ which implies that $(\bigcap_{i=1}^{\infty} N_i)^c = 1_{\sim}$. Then $\bigcup_{i=1}^{\infty} N_i^c = 1_{\sim}$ and $(IBVInt(N_i))^c = 1_{\sim}$. Then we have $IBVCl(N_i^c) = 1_{\sim}$ and hence N_i^c 's are Interval-Valued bipolar vague dense set in (S, IBV_{τ}) . Also $N_i \supseteq N_j^c$ implies that $N_i^c \supseteq (N_j^c)^c$. Let $M_i = N_i^c$. Then we have $\bigcup_{i=1}^{\infty} M_i = 1_{\sim}$, where the Interval-Valued bipolar vague sets M_i 's are Interval-Valued bipolar vague dense set in (S, IBV_{τ}) such that $M_i \subseteq M_j^c$, for $i \neq j$. Therefore (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space.

Proposition 5.5: Let N_i 's be an Interval-Valued bipolar vague nowhere dense set in an Interval-Valued bipolar vague topological space (S, IBV_{τ}) such that $N_i \supseteq N_j^c$, for $i \neq j$. If $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$, then (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space.

Proof: Suppose that $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$, where N_i 's are Interval-Valued bipolar vague nowhere dense set in (S, IBV_{τ}) such that $N_i \supseteq N_j^c$, for $i \neq j$. Now $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$, implies that $(\bigcap_{i=1}^{\infty} N_i)^c = 1_{\sim}$. Then $\bigcup_{i=1}^{\infty} N_i^c = 1_{\sim}$. Since N_i 's are Interval-Valued bipolar vague nowhere dense set in (S, IBV_{τ}) , $IBVInt(IBVCl(N_i)) = 0_{\sim}$. Now $IBVInt(N_i) \subseteq IBVInt(IBVCl(N_i)) = 0_{\sim}$ implies that $IBVInt(N_i) = 0_{\sim}$. Then $(IBVInt(N_i))^c = 1_{\sim}$. Then we have $IBVCl(N_i^c) = 1_{\sim}$ and hence N_i^c 's are Interval-Valued bipolar vague dense set in (S, IBV_{τ}) . Also $N_i \supseteq N_j^c$ implies that $N_i^c \supseteq (N_j^c)^c$. Let $M_i = N_i^c$. Then we have $\bigcup_{i=1}^{\infty} M_i = 1_{\sim}$, where the Interval-Valued bipolar vague sets M_i 's are Interval-Valued bipolar vague dense set in (S, IBV_{τ}) such that $M_i \subseteq M_j^c$, for $i \neq j$. Therefore (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space.

Proposition 5.6: If $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$, where N_i 's are Interval-Valued bipolar vague σ -nowhere dense set in an Interval-Valued bipolar vague topological space (S, IBV_{τ}) such that $N_i \supseteq N_j^c$, for $i \neq j$. Then (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space.

Proof: Suppose that $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$, where N_i 's are Interval-Valued bipolar vague σ -nowhere dense set in (S, IBV_{τ}) such that $N_i \supseteq N_j^c$, for $i \neq j$. Now $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$, implies that $(\bigcap_{i=1}^{\infty} N_i)^c = 1_{\sim}$. Then $\bigcup_{i=1}^{\infty} N_i^c = 1_{\sim}$. Since N_i 's are Interval-Valued bipolar vague σ -nowhere dense set in (S, IBV_{τ}) , by Proposition, N_i^c 's are Interval-Valued bipolar vague dense and Interval-Valued bipolar vague G_{δ} -sets in (S, IBV_{τ}) . Also $N_i \supseteq N_j^c$ implies that $N_i^c \supseteq (N_j^c)^c$. Let $M_i = N_i^c$. Then we have $\bigcup_{i=1}^{\infty} M_i = 1_{\sim}$, where the Interval-Valued bipolar vague sets M_i 's are Interval-Valued bipolar vague dense set in (S, IBV_{τ}) such that $M_i \subseteq M_j^c$, for $i \neq j$. Therefore (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space.

Proposition 5.7: If $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where N_i 's are Interval-Valued bipolar vague open sets such that $N_i \subseteq N_j^c$ for $i \neq j$, in an Interval-Valued bipolar vague hyper connected space in (S, IBV_{τ}) then (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space.

Proof: Let (S, IBV_{τ}) be an Interval-Valued bipolar vague hyper connected space in which $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where N_i 's are Interval-Valued bipolar vague open sets such that $N_i \subseteq N_j^c$ for $i \neq j$. Since (S, IBV_{τ}) is an Interval-Valued bipolar vague hyper connected space, then the Interval-Valued bipolar vague open sets N_i 's are Interval-Valued bipolar vague dense sets in (S, IBV_{τ}) . Therefore (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space.

Proposition 5.8: If the Interval-Valued bipolar vague topological space (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space, then $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where $IBVInt(N_i) = 0_{\sim}$ and the Interval-Valued bipolar vague sets N_i 's are not an Interval-Valued bipolar vague open sets in (S, IBV_{τ}) .

Proof: Let (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space, then by Proposition 5.2, $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague dense set in (S, IBV_{τ}) , such that $IBVInt(N_i) = 0_{\sim}$, suppose that N_i 's are Interval-Valued bipolar vague non-zero open sets in (S, IBV_{τ}) . Then $IBVInt(N_i) = N_i$. This would imply that $IBVInt(N_i) = 0_{\sim}$ in (S, IBV_{τ}) . Therefore, if (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space, then $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where $IBVInt(N_i) = 0_{\sim}$ and the Interval-Valued bipolar vague sets N_i 's are not an Interval-Valued bipolar vague open sets in (S, IBV_{τ}) .

Proposition 5.9: If the Interval-Valued bipolar vague topological space (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space, then (S, IBV_{τ}) is not an Interval-Valued bipolar vague submaximal space.

Proof: Let (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space, then by proposition 5.8, $\bigcup_{i=1}^{\infty} N_i = 1_{\sim}$, where the Interval-Valued bipolar vague sets N_i 's are Interval-Valued bipolar vague dense set in (S, IBV_{τ}) and N_i 's are not Interval-Valued bipolar vague open sets in (S, IBV_{τ}) . Hence the Interval-Valued bipolar vague dense sets N_i 's are not Interval-Valued bipolar vague open sets in (S, IBV_{τ}) . Therefore (S, IBV_{τ}) is not an Interval-Valued bipolar vague submaximal space.

Proposition 5.10: If an Interval-Valued bipolar vague topological space (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space, then $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$, where N_i 's are Interval-Valued bipolar vague dense sets in (S, IBV_{τ}) .

Proof: Suppose that Interval-Valued bipolar vague topological space (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space. Then by proposition 5.3 (S, IBV_{τ}) is an Interval-Valued bipolar vague almost resolvable space. Then we have $\bigcup_{i=1}^{\infty} M_i = 1_{\sim}$, where $IBVInt(M_i) = 0_{\sim}$ in (S, IBV_{τ}) . This implies that $\bigcap_{i=1}^{\infty} (M_i)^c = 0_{\sim}$, where $IBVCl(M_i)^c = 1_{\sim}$. Let $M_i^c = N_i$. Therefore $\bigcap_{i=1}^{\infty} N_i = 0_{\sim}$, where N_i 's are Interval-Valued bipolar vague dense sets in (S, IBV_{τ}) .

Proposition 5.11: If the Interval-Valued bipolar vague topological space (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space, then (S, IBV_{τ}) is an Interval-Valued bipolar vague resolvable space.

Proof: Suppose that Interval-Valued bipolar vague topological space (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -resolvable space, then by proposition 5.2 we have $IBVInt(N_i) = 0_{\sim}$. This implies that $IBVCl(N_i^c) = 1_{\sim}$. Hence (S, IBV_{τ}) is an Interval-Valued bipolar vague resolvable space.

Definition 5.12: An Interval-Valued bipolar vague topological space (S, IBV_{τ}) is said to be an Interval-Valued bipolar vague strongly irresolvable space if $IBVCl(IBVInt(N)) = 1_{\sim}$, for each Interval-Valued bipolar vague dense set N in (S, IBV_{τ}) .

Proposition 5.13: If $IBVInt(N) = 0_{\sim}$ for an Interval-Valued bipolar vague set N in an Interval-Valued bipolar vague strongly irresolvable space (S, IBV_{τ}) , then N is an Interval-Valued bipolar vague nowhere dense set in (S, IBV_{τ}) .

Proof: Let N be an Interval-Valued bipolar vague set in (S, IBV_{τ}) such that $IBVInt(N) = 0_{\sim}$. Then we have $(IBVInt(N))^c = 1_{\sim}$ implies $IBVCl(N^c) = 1_{\sim}$. That is, N^c is an Interval-Valued bipolar vague dense set in (S, IBV_{τ}) . Since (S, IBV_{τ}) is an Interval-Valued bipolar vague strongly irresolvable space, $(IBVCl(IBVInt(N^c))) = 1_{\sim}$ for the Interval-Valued bipolar vague dense set N^c in (S, IBV_{τ}) . Then we have $(IBVCl(IBVInt(N^c)))^c = 0_{\sim}$ implies $IBVInt(IBVCl(N)) = 0_{\sim}$. Therefore, N is an Interval-Valued bipolar vague nowhere dense set in (S, IBV_{τ}) .

Proposition 5.14: If $IBVInt(IBVCl(N)) \neq 0_{\sim}$ for an Interval-Valued bipolar vague set N in an Interval-Valued bipolar vague strongly irresolvable space (S, IBV_{τ}) , then $IBVInt(N) \neq 0_{\sim}$ in (S, IBV_{τ}) .

Proof: Suppose that $IBVInt(N) = 0_{\sim}$ for an Interval-Valued bipolar vague set N in an Interval-Valued bipolar vague strongly irresolvable space in (S, IBV_{τ}) . Then $(IBVInt(N))^c = 1_{\sim}$ implies that $IBVCl(N^c) = 1_{\sim}$. Hence N^c is an Interval-Valued bipolar vague dense set in (S, IBV_{τ}) . Since (S, IBV_{τ}) is an Interval-Valued bipolar vague strongly irresolvable space, $IBVCl(IBVInt(N^c)) = 1_{\sim}$. Then we have $IBVInt(IBVCl(N)) = 0_{\sim}$ which is a contradiction. Hence, we have $IBVInt(N) \neq 0_{\sim}$ in (S, IBV_{τ}) .

Proposition 5.15: If $N_i \subseteq N_j^c$ ($i \neq j$), where N_i is an Interval-Valued bipolar vague dense set in an Interval-Valued bipolar vague strongly irresolvable space (S, IBV_{τ}) , then N_j is an Interval-Valued bipolar vague nowhere dense set in (S, IBV_{τ}) .

Proof: Let N_i be an Interval-Valued bipolar vague dense set in (S, IBV_{τ}) . Since (S, IBV_{τ}) is an Interval-Valued bipolar vague strongly irresolvable space, we have $IBVCl(IBVInt(N_i)) = 1_{\sim}$. Now $N_i \subseteq N_j^c$ ($i \neq j$) implies that $IBVCl(IBVInt(N_i)) \subseteq IBVCl(IBVInt(N_j^c))$ that is $1_{\sim} \subseteq IBVCl(IBVInt(N_j)) = 0_{\sim}$. Therefore we have $IBVCl(IBVInt(N_j^c)) = 1_{\sim}$. Hence $IBVInt(IBVCl(N_j)) = 0_{\sim}$. Therefore N_j is an Interval-Valued bipolar vague nowhere dense set in (S, IBV_{τ}) .

Proposition 5.16: If $N_i \subseteq N_j^c$ ($i \neq j$), where N_i is an Interval-Valued bipolar vague dense set in an Interval-Valued bipolar vague strongly irresolvable space (S, IBV_{τ}) and if $IBVCl(\bigcap_{i=1}^{\infty} N_i) = 1_{\sim}$ then (S, IBV_{τ}) is an Interval-Valued bipolar vague Baire space.

Proof: Let $N_i \subseteq N_j^c$ ($i \neq j$), where N_i is an Interval-Valued bipolar vague dense set in an Interval-Valued bipolar vague strongly irresolvable space (S, IBV_{τ}) . Then by Proposition 5.15, N_j is an Interval-Valued bipolar vague nowhere dense set in (S, IBV_{τ}) . Now $N_i \subseteq N_j^c$ ($i \neq j$) implies that $IBVCl(\bigcap_{i=1}^{\infty} N_i) \subseteq IBVCl(\bigcap_{j=1}^{\infty} N_j^c)$. Then we have $1_{\sim} \subseteq IBVCl(\bigcap_{j=1}^{\infty} N_j^c)$ that is $1_{\sim} \subseteq (IBVInt(\bigcup_{j=1}^{\infty} N_j))^c$. This implies that $IBVInt(\bigcup_{j=1}^{\infty} N_j) \subseteq 0_{\sim}$, that is $IBVInt(\bigcup_{j=1}^{\infty} N_j) = 0_{\sim}$. Hence $IBVInt(\bigcup_{j=1}^{\infty} N_j) = 0_{\sim}$, where N_j 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_{τ}) implies that (S, IBV_{τ}) is an Interval-Valued bipolar vague Baire space.

Proposition 5.17: If (S, IBV_{τ}) is an Interval-Valued bipolar vague Baire space and an Interval-Valued bipolar vague strongly irresolvable space, then $IBVCl(\bigcap_{i=1}^{\infty} M_i) = 1_{\sim}$ where M_i 's are Interval-Valued bipolar vague dense sets in (S, IBV_{τ}) .

Proof: Let (S, IBV_{τ}) be an Interval-Valued bipolar vague Baire space. Then, $IBVInt(\bigcup_{i=1}^{\infty} N_i) = 0_{\sim}$ where N_i 's are Interval-Valued bipolar vague nowhere dense sets in (S, IBV_{τ}) . Then N_i^c 's are Interval-Valued bipolar vague dense sets in (S, IBV_{τ}) . Now $IBVCl(\bigcap_{i=1}^{\infty} M_i) = 0_{\sim}$ implies $IBVCl(\bigcap_{i=1}^{\infty} N_i^c) = 1_{\sim}$. Let $N_i^c = M_i$ then $IBVCl(\bigcap_{i=1}^{\infty} M_i) = 1_{\sim}$ where M_i 's are Interval-Valued bipolar vague dense sets in (S, IBV_{τ}) .

Proposition 5.18: Let the Interval-Valued bipolar vague topological space (S, IBV_τ) be an Interval-Valued bipolar vague strongly irresolvable space then we have the following:

- (i) (S, IBV_τ) is an Interval-Valued bipolar vague almost irresolvable space, then (S, IBV_τ) is an Interval-Valued bipolar vague second category space.
- (ii) (S, IBV_τ) is an Interval-Valued bipolar vague almost resolvable space, then (S, IBV_τ) is an Interval-Valued bipolar vague first category space.

Proof: (i) Let (S, IBV_τ) is an Interval-Valued bipolar vague almost irresolvable space. Then $\bigcup_{i=1}^{\infty} N_i \neq 1_\sim$, where the Interval-Valued bipolar vague sets N_i 's in (S, IBV_τ) are such that $IBVInt(N_i) = 0_\sim$. Now we have $IBVCl(N_i^c) = 1_\sim$ and hence N_i^c 's are Interval-Valued bipolar vague dense sets in (S, IBV_τ) . Since (S, IBV_τ) is an Interval-Valued bipolar vague strongly irresolvable space, for the Interval-Valued bipolar vague dense set N_i , we have $IBVCl(IBVInt(N_i^c)) = 1_\sim$. This implies that $IBVInt(IBVCl(N_i)) = 0_\sim$. Then $\bigcup_{i=1}^{\infty} N_i \neq 1_\sim$, where N_i 's are the Interval-Valued bipolar vague nowhere dense sets in (S, IBV_τ) , implies that (S, IBV_τ) is an Interval-Valued bipolar vague second category space.

(ii) The proof is obvious.

Definition 5.19: An Interval-Valued bipolar vague topological space (S, IBV_τ) is called an Interval-Valued bipolar vague almost \mathcal{P} -space if for every non-zero Interval-Valued bipolar vague G_δ -set N in (S, IBV_τ) , $IBVInt(N) \neq 0_\sim$ in (S, IBV_τ) .

Proposition 5.20: If each non-zero Interval-Valued bipolar vague G_δ -set is an Interval-Valued bipolar vague dense set in an Interval-Valued bipolar vague strongly irresolvable space, then (S, IBV_τ) is an Interval-Valued bipolar vague almost \mathcal{P} -space.

Proof: Let A be a non-zero Interval-Valued bipolar vague dense and an Interval-Valued bipolar vague G_δ -set in an Interval-Valued bipolar vague strongly irresolvable space in (S, IBV_τ) . Then we have $IBVCl(IBVInt(N)) = 1_\sim$ in (S, IBV_τ) and thus we have $IBVInt(N) \neq 0_\sim$. (Otherwise if $IBVInt(N) = 0_\sim$, then $IBVCl(IBVInt(N)) = IBVCl(0_\sim) \neq 1_\sim$, a contradiction). Hence, for every non-zero Interval-Valued bipolar vague dense and Interval-Valued bipolar vague G_δ -set N in (S, IBV_τ) , we have $IBVInt(N) \neq 0_\sim$. Therefore (S, IBV_τ) is an Interval-Valued bipolar vague almost \mathcal{P} -space.

Proposition 5.21: If the Interval-Valued bipolar vague topological space (S, IBV_τ) is an Interval-Valued bipolar vague strongly irresolvable space, then (S, IBV_τ) is an Interval-Valued bipolar vague almost $G\mathcal{P}$ -space.

Proof: The Proof is obvious.

Proposition 5.22: If the Interval-Valued bipolar vague topological space (S, IBV_τ) is an Interval-Valued bipolar vague strongly irresolvable and Interval-Valued bipolar vague Baire space then for any Interval-Valued bipolar vague first category set N in (S, IBV_τ) , then N is an Interval-Valued bipolar vague nowhere dense set in (S, IBV_τ) .

Proof: Let N be an Interval-Valued bipolar vague first category set in (S, IBV_τ) . Since (S, IBV_τ) is an Interval-Valued bipolar vague Baire space, by proposition, $(IBVInt(N)) = 0_\sim$ in (S, IBV_τ) . Then we have $(IBVInt(N))^c = 1_\sim$. This implies that $IBVCl(N^c) = 1_\sim$, since (X, IBV_τ) is an Interval-Valued bipolar vague strongly irresolvable space, for the Interval-Valued bipolar vague dense set A^c in (S, IBV_τ) , we have $IBVCl(IBVInt(N^c)) = 1_\sim$ implies $(IBVInt(IBVCl(N)))^c = 1_\sim$ and hence $IBVInt(IBVCl(N)) = 0_\sim$. Therefore, A is an Interval-Valued bipolar vague nowhere dense set in (S, IBV_τ) .

Proposition 5.23: Let (S, IBV_τ) be an Interval-Valued bipolar vague topological space. Then the following are equivalent:

- (i) (S, IBV_τ) is an Interval-Valued bipolar vague σ -Baire space.
- (ii) $IBVInt(N) = 0_\sim$ for every Interval-Valued bipolar vague σ -first category set N in (S, IBV_τ) .
- (iii) $IBVCl(N) = 1_\sim$ for every Interval-Valued bipolar vague σ -residual set N in (S, IBV_τ) .
- (iv)

Proof: (i) \Rightarrow (ii). Let N be an Interval-Valued bipolar vague σ -first category set N in (S, IBV_τ) . Then $N = \bigcup_{i=1}^{\infty} N_i$ where N_i 's are Interval-Valued bipolar vague σ -nowhere dense set in (S, IBV_τ) . Then we have $IBVInt(N) = IBVInt(\bigcup_{i=1}^{\infty} N_i)$. Since (S, IBV_τ) is an Interval-Valued bipolar vague σ -Baire space and $IBVInt(\bigcup_{i=1}^{\infty} N_i) = 0_\sim$. Hence $IBVInt(N) = 0_\sim$ for any Interval-Valued bipolar vague σ -first category set N in (S, IBV_τ) .

(ii) \Rightarrow (iii). Let N be an Interval-Valued bipolar vague σ -residual set in (S, IBV_{τ}) . Then N^c is an Interval-Valued bipolar vague σ -first category in (S, IBV_{τ}) . By hypothesis, $IBVInt(N^c) = 0_{\sim}$. Hence $(IBVCl(N)) = 1_{\sim}$, for any Interval-Valued bipolar vague σ -residual set S in (S, IBV_{τ}) . \Rightarrow (i). Let N be an Interval-Valued bipolar vague σ -first category set N in (S, IBV_{τ}) . Then $N = \bigcup_{i=1}^{\infty} N_i$ where N_i 's are Interval-Valued bipolar vague σ -nowhere dense set in (S, IBV_{τ}) . This implies that N^c is an Interval-Valued bipolar vague σ -residual set in (S, IBV_{τ}) . By hypothesis, we have $IBVCl(N^c) = 1_{\sim}$. Hence we have $IBVInt(N) = 0_{\sim}$. That is $IBVInt(\bigcup_{i=1}^{\infty} N_i) = 0_{\sim}$ where N_i 's are Interval-Valued bipolar vague σ -nowhere dense set in (S, IBV_{τ}) . Hence (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -Baire space.

Proposition 5.24: If the Interval-Valued bipolar vague topological space (S, IBV_{τ}) is an Interval-Valued bipolar vague strongly irresolvable and Interval-Valued bipolar vague σ -Baire space and N is an Interval-Valued bipolar vague σ -first category set in (S, IBV_{τ}) , then N is an Interval-Valued bipolar vague nowhere dense set in (S, IBV_{τ}) .

Proof: Let N be an Interval-Valued bipolar vague σ -first category set and Interval-Valued bipolar vague σ -Baire space in (S, IBV_{τ}) . Then by Proposition 5.23, $IBVInt(N) = 0_{\sim}$ in (S, IBV_{τ}) , which implies that $IBVCl(N^c) = 1_{\sim}$. Since (S, IBV_{τ}) is an Interval-Valued bipolar vague strongly irresolvable for an Interval-Valued bipolar vague dense set N^c in (S, IBV_{τ}) , we have $IBVCl(IBVInt(N^c)) = 1_{\sim}$ and hence $IBVInt(IBVCl(N)) = 0_{\sim}$. Therefore, N is an Interval-Valued bipolar vague nowhere dense set in (S, IBV_{τ}) .

Proposition 5.25: If N_i 's are Interval-Valued bipolar vague σ -first category sets in an Interval-Valued bipolar vague strongly irresolvable and an Interval-Valued bipolar vague σ -Baire space in (S, IBV_{τ}) then $\bigcup_{i=1}^{\infty} N_i$ is an Interval-Valued first category set in (S, IBV_{τ}) .

Proof: Let N_i 's be an Interval-Valued bipolar vague σ -first category sets in an Interval-Valued bipolar vague strongly irresolvable and an Interval-Valued bipolar vague σ -Baire space in (S, IBV_{τ}) . Then by Proposition 5.24, N_i 's are Interval-Valued bipolar vague nowhere dense set in (S, IBV_{τ}) and hence $\bigcup_{i=1}^{\infty} N_i$ is an Interval-Valued first category set in (S, IBV_{τ}) .

Proposition 5.26: If $IBVInt(\bigcup_{i=1}^{\infty} N_i) = 0_{\sim}$ where N_i 's are Interval-Valued bipolar vague σ -first category sets in an Interval-Valued bipolar vague strongly irresolvable and an Interval-Valued bipolar vague σ -Baire space in (S, IBV_{τ}) then (S, IBV_{τ}) is an Interval-Valued Baire space.

Proof: Let N_i 's are Interval-Valued bipolar vague σ -first category sets in an Interval-Valued bipolar vague strongly irresolvable and an Interval-Valued bipolar vague σ -Baire space in (S, IBV_{τ}) . Then by Proposition 5.25, $\bigcup_{i=1}^{\infty} N_i$ is an Interval-Valued first category set in (S, IBV_{τ}) . Then N_i 's are Interval-Valued bipolar vague nowhere dense set in (S, IBV_{τ}) . Since (S, IBV_{τ}) is an Interval-Valued bipolar vague σ -Baire space we have $IBVInt(\bigcup_{i=1}^{\infty} N_i) = 0_{\sim}$ where N_i 's are Interval-Valued bipolar vague nowhere dense set in (S, IBV_{τ}) . Hence the Interval-Valued bipolar vague topological space (S, IBV_{τ}) is an Interval-Valued Baire space.

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